Hwk # 3 MAT 5150 Prof. M. Alvo

6.2 $f_{X_1,\dots,X_n}(x_1 \dots x_n) = e^{\theta \Sigma i - \Sigma x_i} \cdot \prod_{i=1}^{n} I_{[i\theta,\infty)}(x) = e^{\theta \Sigma i - \Sigma x_i} \prod_{1}^{n} I_{[\theta,\infty)}(x_i/i)$

on 3 $= \left[\theta^{\Sigma i} I_{[\theta,\infty)}(\min x_i/i)\right] e^{-\Sigma x_i}$

By the Factorization Theorem, $\min X_i/i$ is sufficient

6.3 $f_{X_1 \dots X_n}(x_1 \dots x_n) = \frac{e^{-\Sigma(x_i - \mu)/\sigma}}{\sigma^n}, \; I_{(\mu,\infty)}(\min x_i)$

on 3 Hence $(\min X_i, \Sigma X_i)$ is sufficient for (μ, σ).

6.5 $f_{X_1 \dots X_n}(x_1 \dots x_n) = \left(\prod_1^n \frac{1}{2i}\right)\left(\frac{1}{\theta^n}\right) \prod_1^n I_{(\theta-1,\theta+1)}(x_i/i)$

on 3 $= \left(\prod^n \frac{1}{2i}\right)\left(\frac{1}{\theta^n}\right) \prod \left[I_{(\theta-1,\infty)}(x_i/i) \; I_{(-\infty,\theta+1)}(x_i/i)\right]$

$= \left(\prod^n \frac{1}{2i}\right)\left(\frac{1}{\theta^n}\right) I_{(\theta-1,\infty)}(\min x_i/i) \; I_{(-\infty,\theta+1)}(\max x_i/i)$

Hence $(\min x_i/i, \max x_i/i)$ is sufficient for θ.

on 3 6.6 $(\prod x_i, \Sigma x_i)$ is sufficient for (α, β); on 3 6.7 $(X_{(1)} \dots X_{(n)})$ is sufficient.

6.9 a) $\bar{X}$ is minimal sufficient since $\frac{f(x_1 \dots x_n)}{f(y_1 \dots y_n)} = e^{-(\Sigma x^2 - \Sigma y^2)/2} \cdot e^{n(\bar{x}-\bar{y})\theta}$

is independent of θ if and only if $\bar{x} = \bar{y}$

on 5 b) $\min X_i$ is minimal sufficient since $\frac{f(x_1 \dots x_n)}{f(y_1 \dots y_n)} = e^{-(\Sigma x - \Sigma y)} \cdot \frac{I_{(\theta,\infty)}(\min x_i)}{I_{(\theta,\infty)}(\min y_i)}$

is independent of θ if and only if $\min x_i = \min y_i$

c) $\frac{f(x_1 \dots x_n)}{f(y_1 \dots y_n)} = e^{-(\Sigma x - \Sigma y)} \cdot \frac{\prod\left(1 + e^{(x_i - \theta)}\right)^2}{\prod\left(1 + e^{(y_i - \theta)}\right)^2}$

Both numerator and denominator are polynomials in $e^{-\theta}$ of degree $2n$. The ratio is independent of θ if they have the same roots. Hence, the $(x_1 \dots x_n)$ must be some permutation of $(y_1, \dots, y_n)$. Hence the order statistics are sufficient i.e. $(X_{(1)}, \dots, X_{(n)})$ is sufficient for θ.

d) same as c) where the roots are of the form $x_i \pm i$ and $y_i \pm i$ respectively

e) For minimal sufficiency we need $\Sigma|x_i - \theta| = \Sigma|y_i - \theta|$ which implies $(X_{(1)}, \dots, X_{(n)})$ is min. sufficient